The unequal arm length is the source of measurement error which rises with the magnitude of the reflection-coefficient change of the measured resonator. It may be removed by the accurate production of the hybrid T or by correction of measured data. The value of the correction factor may be established by measuring the output signal of the MRB, if both resonators are tuned outside the chosen working frequency band. In this case, resonators represent short circuits at the ends of the througharm of the magic T with reflection coefficients $\Gamma_2 = \Gamma_3 = -1$, and detuned $|b_4|/|a_1| = 0.5\Theta$.

3) The frequency dependence of the S-parameters of the magic T is shown as follows. In analysis of the theoretical model of the MRB it was presumed that the elements of the scattering matrix are constant and real over the entire frequency band. This presumption may not be satisfied with actual components, however, and hence the derived relations may not have accurate validity. The analytical expression of the S-parameter dependence of the hybrid junction on frequency, and hence the investigation of the influence of the frequency changes upon the accuracy of the method is not a simple matter, in general. Experimentally, two procedures are possible: a) to measure, within the selected frequency range, the parameters of the magic T and to apply them in deriving new relations analogically to the procedure explained in the theoretical section; b) to choose, on the basis of measurement in a broader frequency range, a suitable working frequency range in which the scattering parameters satisfy the presumptions of the derived theoretical model.

References

[1] M. Sucher, "Measurement of Q," in *Handbook of Microwave Measurements*, M. Sucher, and J. Fox, Ed., vol. 2. New York: Wiley, 1963, pp. 417–493

[2] H. E. Bussey, "Measurement of RF properties of materials: A survey," *Proc. IEEE*, vol. 55, pp. 1046–1053, June 1967.

[3] E. L. Ginzton, *Microwave Measurements.* New York: McGraw-Hill, 1957, ch. 10, pp. 435–452.

[4] R. A. Waldron, *Theory of Guided Electromagnetic Waves.* London: Van Nostrand, 1969, ch. VI, pp. 292–318.

Comparison Method of Measurement Q of Microwave Resonators

IVAN KNEPPO

***Abstract*—A method for the measurement of the quality factor of microwave resonators is given. It is based on the comparison of the transmitted power of the measured resonator with that of a reference resonator. The characteristics obtained by simultaneous display of the transmitted powers of the two resonators in Cartesian coordinates are analyzed. The relationship between the shape of the resulting curve and the parameters of the reference and measured resonators is discussed. The principal scheme of the X-band measuring microwave set and the measurement procedure are described. Some verification of the measurement results are presented. The method described is especially convenient for measuring small relative changes in the quality factor. It can be utilized for measuring the microwave loss tangent of materials by perturbation resonator methods.**

Nomenclature List

i	Integer denoting the arm of the microwave network.
K	Ratio of the gain increase of the vertical amplifier.
$P_i(\omega)$	Transmitted power of the ith resonator.
$P_i(\omega_{0i})$	Transmitted power of the ith resonator at resonance.
Q_{Li}	Loaded quality factor of the ith resonator.
x, y	Real variables in the Cartesian coordinates.
x_1	Point of the extreme.
δ	Difference function.
δ_1	Value of the extreme of the difference function.
θ	Relative difference of resonance frequencies.
ν	Relative change of the frequency.
ω	Frequency.
ω_{0i}	Resonance frequency of the ith resonator.

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I. Introduction

The most widely used method of measuring microwave cavity Q is the measurement of the bandwidth of the resonator. A number of bandwidth-measurement methods have been evolved based on the accurate measurement of the frequency interval between the half-power points of a resonance curve, or between the inflection points, or based on the phase shift of the modulation envelope of the transmitted signal. A detailed description of these methods is given in a number of publications; a good survey is in [1]. A common deficiency of the bandwidth methods is the difficulty in measuring small changes in the cavity Q factor which occur, for example, in perturbation methods for the measurement of the loss tangent of low-loss dielectrics. It is also a significant fact that such a measuring set, capable of detecting small changes in the cavity Q factor, is rather sophisticated, and microwave measuring instruments belonging to the highest precision class are expensive. Especially high demands are laid upon the frequency stability of the microwave generator and upon the precision of the frequency meter which is usually a digital one.

Less widespread methods, in microwave practice, bearing upon the measurement of the cavity Q factor are those based on the comparison of the measured resonator with a reference resonator, or with a reference resonance circuit with known parameters. These methods are convenient in those cases when accuracy in measuring the absolute value of the Q factor is secondary and the primary requirement is to indicate and measure the small relative change in the resonator parameters. Papers [2] and [3] present a description of a method by which the resonance curve of the measured microwave resonator is compared with that of a calibrated lower frequency resonance circuit of variable Q. Paper [4] discusses the comparison of the measured quality factor to that of the reference resonator by means of the microwave resonator bridge.

In the present short paper, a new comparison method for measuring the microwave resonator quality factor is suggested. The proposed method is based on the comparison of the powers transmitted through the measured and reference resonators. The transmitted powers of both resonators are displayed, during the sweeping, on the oscilloscope screen in Cartesian coordinates with the transmitted power of the reference resonator recorded on the x axis while the difference of the transmitted powers of the measured and reference resonators is recorded on the y axis. The shape of the curve displayed depends on the difference in the parameters of both resonators, and expressively varies with small changes in the parameters. The tuning of the measured resonator to the resonance frequency of the reference resonator is indicated very markedly. Also it is possible to read from the graph of the resultant curve all the data which are necessary for the calculation of the cavity quality factor. The proposed method is very convenient for the measurement of small changes in the quality factor and resonant frequency of the microwave resonator.

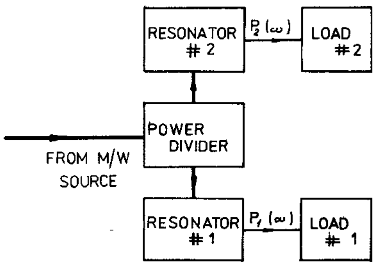

Fig. 1. Arrangement of two resonators in the output arms of the power divider.

II. Theory

Let us consider two resonators of the transmission type located at the output arms of a power divider, as shown in Fig. 1, and assume a power divider that is balanced in the whole frequency range, its outputs being mutually isolated (hybrid divider). Assume further that the coupling structures of the resonators are essentially lossless, interact negligbly with each other, and that the output line of each resonator is matched. Then the power delivered to the load through the individual resonator is

$$P_i(\omega) = P_i(\omega_{0i}) \left[1 + Q_{Li}^2 \left(\frac{\omega}{\omega_{0i}} - \frac{\omega_{0i}}{\omega}\right)^2\right]^{-1} \quad (1)$$

for $i = 1, 2$, where ω_{0i} are the resonance frequencies and Q_{Li} are the loaded quality factors of the resonators. $P_i(\omega_{0i})$ represent the transmitted power at resonance.

When the transmitted powers of both resonators are displayed in a Cartesian coordinate system in the way that $x \equiv P_1(\omega)/P_1(\omega_{01})$ and $y \equiv P_2(\omega)/P_2(\omega_{02})$, we get the curve with parametric equations

$$x = \left[1 + Q_{L1}^2 \left(\frac{\omega}{\omega_{01}} - \frac{\omega_{01}}{\omega}\right)^2\right]^{-1} \quad (2a)$$

$$y = \left[1 + Q_{L2}^2 \left(\frac{\omega}{\omega_{02}} - \frac{\omega_{02}}{\omega}\right)^2\right]^{-1} \quad (2b)$$

defined in the intervals $0 \le x \le 1$, $0 \le y \le 1$. If the resonance frequencies of both resonators are mutually slightly shifted and the curve is analyzed only in the vicinity of the resonance, then (2a) and (2b) will be simplified

$$x \doteq (1 + 4Q_{L1}^2\nu^2)^{-1} \quad (3a)$$

$$y \doteq (1 + 4Q_{L2}^2(\nu - \theta)^2)^{-1} \quad (3b)$$

where $\nu = \Delta\omega/\omega_{01}$ is a change of frequency relative to the resonance frequency of the resonator 1, and $\theta = (\omega_{02} - \omega_{01})/\omega_{01}$ is a relative difference of resonance frequencies of both resonators. By elimination of the parameter ν from (3a) and (3b), one gets the explicit equation of the curve in the xy coordinate system, i.e.,

$$y = \left\{1 + \left[\frac{Q_{L2}}{Q_{L1}}\left(\pm\sqrt{\frac{1}{x} - 1} - 2Q_{L1}\theta\right)\right]^2\right\}^{-1}. \quad (4)$$

From analysis of (4) it follows that if $\theta \neq 0$, the equation is ambiguous and the displayed curve is sling shaped. The sense of circulation around the curve depends on the mutual position of the resonances of both resonators. If $\omega_{01} < \omega_{02}$ ($\omega_{01} > \omega_{02}$), then the sense of circulation is counterclockwise (clockwise), when sweeping from lower to higher frequencies. For the case

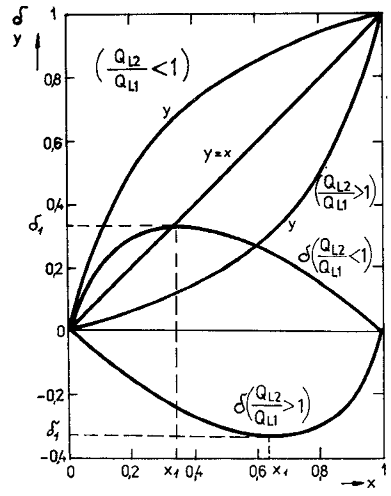

Fig. 2. A typical graph of the function y and differential function δ if the resonances of both resonators are equal.

when the resonances of both resonators are equal $\theta = 0$, the equation of the curve is

$$y = \left[1 + \left(\frac{Q_{L2}}{Q_{L1}}\right)^2 \left(\frac{1}{x} - 1\right)\right]^{-1} \quad (5)$$

which represents the unambiguous function. The ratio of the loaded Q's is a parameter of the curve. If $Q_{L2}/Q_{L1} < 1$ or $Q_{L2}/Q_{L1} > 1$, the curve is concave or convex. If $Q_{L2}/Q_{L1} = 1$, the curve degenerates to the line $y = x$, as shown in Fig. 2. The difference function $\delta = y - x$ has at the point

$$x_1 = \frac{Q_{L2}/Q_{L1}}{1 + Q_{L2}/Q_{L1}}$$

the extreme

$$\delta_1 = \frac{1 - Q_{L2}/Q_{L1}}{1 + Q_{L2}/Q_{L1}}. \quad (6)$$

It is possible to determine the value of δ_1 experimentally and with the aid of (6) to calculate the ratio of loaded Q's, i.e.,

$$\frac{Q_{L2}}{Q_{L1}} = \frac{1 - \delta_1}{1 + \delta_1}. \quad (7)$$

III. Measurement

The experimental setup for measuring the cavity quality factor by the comparison method is shown in Fig. 3. The measuring set consists of an amplitude-leveled microwave sweep generator connected through the isolator and level attenuator to the hybrid power divider. The power divider must have a balanced power division, constant insertion loss, low and constant SWR, and high isolation between the output arms. A magic T was used with isolation > 25 dB and SWR < 1.30 in the frequency range from 8.7 to 9.5 GHz. At each output arm of the power divider a transmission-type resonator is located, each terminated with the matched power sensor of a microwave power meter. One is the test resonator, and its resonance frequency and quality are unknown; the second is a reference resonator with a calibrated dial and a known loaded quality factor. The characteristics of the applied power meters must be linear. The output signals of both power meters are fed into the horizontal amplifier of the oscilloscope and differential amplifier. The output of the differential amplifier is connected to the vertical amplifier of the oscilloscope. The microwave set is completed with two level attenuators located

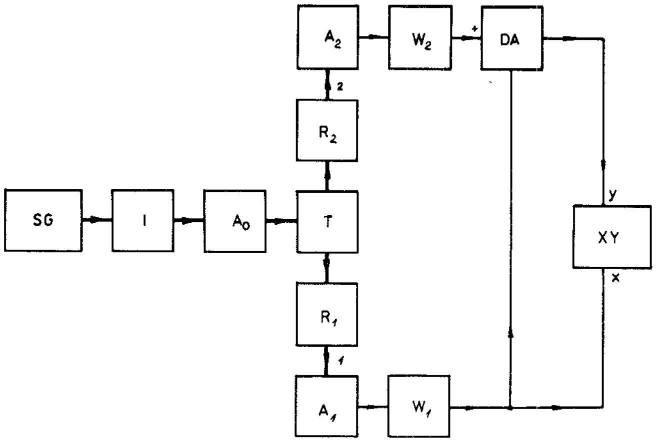

Fig. 3. Block diagram of the measuring set: SG, microwave sweep generator; I, isolator; A_0, level attenuator; T, hybrid power divider; R_1, R_2, resonators; A_1, A_2, attenuators; W_1, W_2, microwave power meters; DA, differential amplifier; XY, oscilloscope.

at the two output arms of the power divider. By sweeping the frequency of the microwave generator in the vicinity of the resonances of both resonators, the display reflects the curves corresponding to the difference function δ. The relative difference of the cavities resonance frequencies is the parameter of the curves displayed and, in accordance with theoretical analyses, the curves are sling shaped when resonances are different and an unambiguous curve is displayed only when both resonators have the same resonance frequency.

The procedure for the comparison measurement of the microwave resonator quality consists of the following steps.

1) Adjust the resonator R_1 until its resonance lies approximately in the center of the sweeping band. On the screen of the oscilloscope the line $y = -x$ is displayed, see Fig. 4(a). During this step the attenuator A_2 is set at maximum attenuation.

2) Read the value y_1 from the screen (in centimeters). This length corresponds to the unity on the y axis, $1 \equiv y_1$ [cm].

3) Lower the attenuation of A_2.

4) Tune the cavity R_2 so that its resonance coincides with the resonance frequency of the cavity R_1, and an unambiguous curve on the screen is displayed, Fig. 4(b).

5) Adjust the variable attenuator A_2 so that the end point A of the curve coincides with the point (1.0) on the x axis, see Fig. 4(c).

6) Adjust the gain of the vertical amplifier of the oscilloscope until the displayed curve covers the whole vertical side of the screen. This step is convenient in increasing the sensitivity of the method. The unity of the y axis is $1/K$ now, where K is the ratio of gain increase.

7) Repeat steps 4 and 5, in order to correct inaccuracy of previous adjustments.

8) Read the value of the curve height y_2 from the display. The value of the extreme of the differential function is $\delta_1 = y_2/K$.

9) Compute the ratio of the loaded quality factors Q_{L2}/Q_{L1} in accordance with (7) and finally also the quality factor of the unknown resonator.

The quality of a TE_{011} resonator was measured by the described comparison method in the X band. A cavity wavemeter of the same type was used as a reference resonator, with a loaded quality factor $Q_{L1} = 11\,200$. The measured value of the maximum of the difference function was $\delta_1 = 0.186 \pm 0.009$, which gives, for the ratio of loaded Q's, $Q_{L2}/Q_{L1} = 0.686 \pm 0.012$ and loaded quality of the measured resonator $Q_{L2} = 7683 \pm 148$. The loaded Q of the measured resonator was, for comparison, also measured by the conventional frequency-marker method from the transmission curve width [1], its value being $Q_{L2} = 7900 \pm 395$. The experiment indicated that the comparison method renders results with a smaller dispersion than the conventional method. The measuring accuracy by the comparison method depends upon the accuracy of the value of the loaded Q of the reference cavity, by the short-term stability of the microwave-generator power level, and by the stability of the gain of the differential amplifier. The long-term unstability influence may be lowered by the repetition of calibration steps 1) and 2). Other sources of measuring errors are: the finite value of isolation between the output arms of the microwave power divider, mismatch of the power sensors to the resonator's output lines, and the different phase characteristics of the vertical and horizontal amplifier of the oscilloscope.

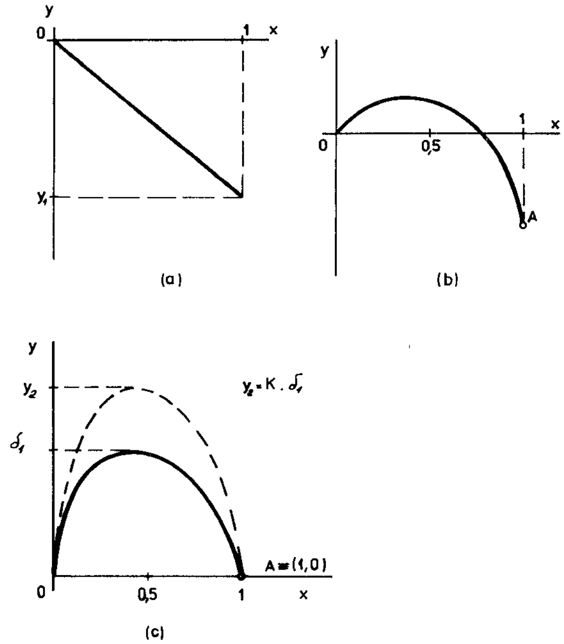

Fig. 4. Typical curves displayed on the oscilloscope screen by the measuring steps.

IV. Conclusions

The method of comparing the resonance curves of the measured and reference resonators in the XY plane is convenient for microwave cavity Q measuring, especially when small changes in the Q are to be measured. The described method is very suitable for microwave loss tangent measurement of low-loss materials. The advantages of the method include the following.

1) High sensitivity to small changes in the measured resonator parameters.

2) Distinct indication of the tuning of the measured resonator on the reference-resonator resonance frequency.

3) Immunity to the parasitic FM of the microwave signal.

4) It is not necessary to fix the half-power level of the signal transmitted through the resonator and to measure precisely the frequencies of the half points.

5) The measuring set is simple.

On the other hand, the need to know of the loaded Q of the reference resonator and the requirement of having two microwave milliwattmeters with linear characteristics lowers the advantages listed.

References

[1] M. Sucher, "Measurement of Q," in *Handbook of Microwave Measurements*, vol. 2, M. Sucher and J. Fox, Ed. Brooklyn, NY: Polytechnic Press, 1963, ch. VIII, pp. 417–493.

[2] H. LeCaine, "The Q of a microwave cavity by comparison with a calibrated high frequency circuit," *Proc. IRE*, vol. 40, pp. 155–157, Feb. 1952.

[3] V. M. Vakhnin and T. F. Kolodina, "Setup for measuring attenuation of the H_{01} wave in short waveguide sections by use of the resonance cavity method," *Radiotechnik. Electronik.*, no. 12, pp. 1486–1491, 1956.

[4] I. Kneppo, L. Nagy, and M. Weis, "The resonator bridge method of the microwave loss tangent measurement," *Elektrotechnický časopis*, vol. 26, pp. 338–343, Apr. 1975.

A Gaussian-Beam Launcher for Microwave Exposure Studies

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***Abstract*—A practical method of producing a focused-microwave exposure field in biological experiments, for selective partial-body irradiation, is described. The proposed structure consists of a dielectric sphere placed in front of, but displaced from, the open end of a corrugated pipe with quarter-wave teeth, carrying the HE_{11} mode. It is shown that this launcher produces a near-circular Gaussian beam in the proximity of the dielectric sphere, with a high on-axis gain factor. Theoretical expressions are derived for the EM fields of the focused beam-wave, and experimental results obtained from a practical launcher confirm the theoretical calculations made.**

Introduction

In the areas of biological researches and medical applications of microwaves, it may be desirable to focus microwave energy in a very small region, close to the focusing lens, for localized exposure of biological subjects. For example, a noncontact selective heating of diseased tissues as an alternative to surgical removal and for selective heating of wounded tissues in conjunction with chemotherapy, need a practical method of launching of a microwave Gaussian beam.

Recently, the theoretical calculations and electric-field measurements made by Ho *et al.* [1] to investigate the focusing effects of plane-wave irradiated dielectric spheres (lenses) of different diameters and dielectric properties indicated the possibility of obtaining a focused spot close to the dielectric sphere. However, the experimental arrangement described in [1], requires a plane-wave source, and hence an anechoic chamber as well as a high-power microwave generator. Hence, to increase the practical utility of the dielectric sphere for microwave irradiation, it is necessary to replace the incident plane wave with a more practical source.

A method of getting a focused microwave beam by means of a launcher (Fig. 1) consisting of a homogeneous dielectric sphere illuminated by a corrugated cylindrical waveguide aperture (scalar horn) is described. The corrugated pipe has quarter-wave teeth and carries the hybrid (HE_{11}) mode. The field distribution at the aperture of the corrugated pipe is used as the sphere-illuminating source and the dielectric sphere is placed in front of, but displaced from, this aperture. The waveguide-to-sphere offset can be chosen to be the optimum value for which the input VSWR is a minimum [2], [3]. The corrugated pipe and the offset dielectric sphere can be conveniently supported by means

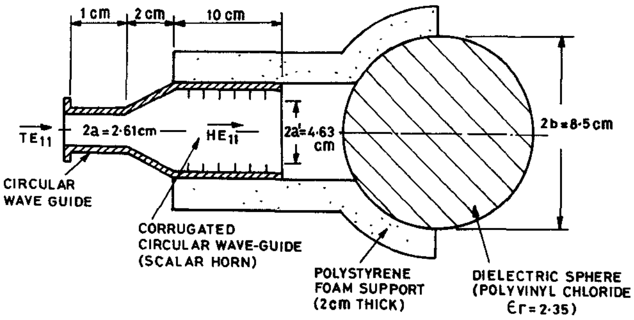

Fig. 1. The test Gaussian-beam launcher. Corrugation parameters: Pipe inner diameter ($2a'$) = 46.3 mm; depth of corrugation (l) = 8.45 mm; groove width (g) = 2.0 mm; tooth width (t) = 0.2 mm (thin blades); and corrugation density equals 15 teeth per wavelength (approx.)

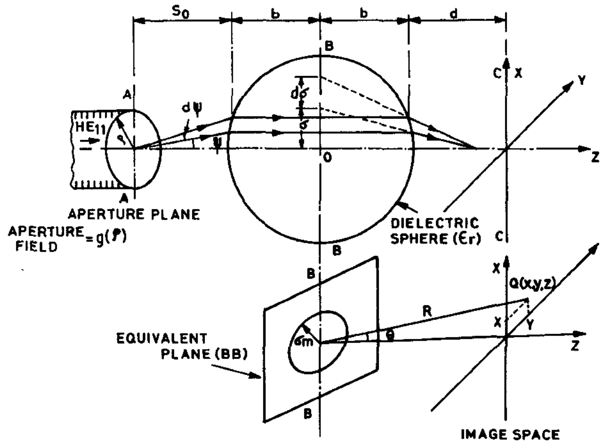

Fig. 2. Optical system showing the merdional section of the dielectric sphere. AA: Aperture plane of corrugated pipe with a symmetrical-field distribution $g(\rho)$, which illuminates the sphere. BB: Equivalent plane through the center of spherical lens with circular aperture of radius σ_m. CC: An XY plane at a distance d behind the dielectric sphere.

of a noninterfering, polystyrene-foam holder, as illustrated in Fig. 1. The purpose of the present work is to demonstrate the feasibility of obtaining a Gaussian (focused) beam in the proximity of the dielectric sphere with the system just described; and the results concerning the on-axis gain factor, etc., of the present launcher are compared with those of the system due to Ho *et al.* [1].

Theoretical Formulation

In [2] and [3], Neelakantaswamy and Banerjee presented a theoretical method to study the radiation behavior of a dielectric sphere kept offset in front of a waveguide aperture. This method is currently extended to analyze the beam wave behind the dielectric sphere of the arrangement depicted in Fig. 1. Referring to the optical system shown in Fig. 2, the approximate diffracted vertical- and horizontal-field components behind the sphere may be denoted as $U_\theta(\theta,R)$ and $U_\phi(\theta,R)$, respectively. The explicit expressions for these functions are given by [2, eqs. (1)–(4)] with the aberration function $V(\sigma)$ being replaced by $V'(\sigma)$, where $V'(\sigma)$ is given by

$$V'(\sigma) = V(\sigma) + \frac{b^2\sigma^2}{2z}. \tag{1}$$

Considering the field distribution at the aperture of the corrugated pipe (AA) carrying the hybrid HE_{11} mode, it is well known that [4]

$$g(\rho) = E_0 J_0(\alpha\rho) \tag{2}$$

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